

Stability of Palatini-f(R) cosmology

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Abstract

The evolution of linear cosmological perturbations in modified theories of gravity is investigated assuming the Palatini formalism. It has been discussed about the stability problem in this model based on the equivalence between f(R) gravity and the scalar tensor theory. However, we study this problem in the physical frame where the matter is minimally coupled. In general, the stability of the superhorizon metric evolution depends on models. We show that the deviation from the superhorizon metric evolution is null for a specific choice for the nonlinear Einstein-Hilbert action, $f(\hat{R}) \sim \hat{R}^n$, where $n \neq 0, 2, 3$. Thus the stability of metric fluctuation is guaranteed in these models. We also study the matter density fluctuation in the general gauge and show the differential equations in super and sub-horizon scales.

1 Introduction

The discovery of the present accelerated expansion of the Universe [1, 2] can be accounted for either by the existence of a homogeneous component of energy with a negative pressure, dubbed dark energy [3], or by a modification of gravitational action with a general function of the scalar curvature instead of the standard Einstein-Hilbert term, named $f(R)$ gravity [4].

Alternative field equations depend not only on the choice of action but also on the variational principles [5]. The Palatini formalism where the metric and the connections are treated as independent variables and the energy momentum tensor does not depend on the independent connection leads to a different theory from what is obtained from the metric formalism. The Palatini formalism of $f(R)$ gravity results in second order differential equations due to the algebraic relation between the curvature scalar and the trace of the energy momentum tensor.

Cosmological background solutions have been studied for various gravitational Lagrangian in the Palatini formalism [6]. Their validity as cosmological theories have been tested for observation [7]. Also the cosmological perturbations have been investigated [8].

It has been known that the Palatini formalism to $f(R)$ gravity is stable for the curvature scalar perturbation [9]. The Newtonian limit of models using the Palatini variational principle gives contradicting results [10]. However, the approach to this problem is using the equivalence between $f(R)$ gravity and scalar-tensor theory. We will use the evolution of linear perturbations in $f(R)$ models in the physical frame as already done in the metric formalism [11].

We study the evolution of linear perturbations in Palatini $f(R)$ gravity in the physical frame where the matter is minimally coupled. We consider the stability of metric fluctuations at high curvature to see the agreement with high redshift cosmological observations. We also investigate the evolution of the matter density fluctuation.

In the next section we review the Palatini $f(R)$ gravity. We investigate the linear perturbation of $f(R)$ gravity in the section III. Compared with previous works [8], we do not specify the gauge to find the matter density fluctuation. In section IV, we derive the stability equation of metric fluctuations in the high curvature limit and show the stability in a specific model. We show the evolutions of the metric fluctuation and the density contrast in the superhorizon and the subhorizon scales in section V. We reach our conclusions in section VI. We also provide the detail calculation in the appendix.

2 Palatini f(R) gravity

We consider a modification to the Einstein-Hilbert action assuming the Palatini formalism, where the metric $g_{\mu\nu}$ and the torsionless connection $\hat{\Gamma}_{\mu\nu}^\alpha$ are independent quantities and the matter action depends only on metric

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} f(\hat{R}(g_{\mu\nu}, \hat{\Gamma}_{\mu\nu}^\alpha)) + \mathcal{L}_m(g_{\mu\nu}, \psi) \right], \quad (2.1)$$

where ψ are matter fields. Then the Ricci tensor is defined solely by the connection

$$\hat{R}_{\mu\nu} = \hat{\Gamma}_{\mu\nu, \alpha}^\alpha - \hat{\Gamma}_{\mu\alpha, \nu}^\alpha + \hat{\Gamma}_{\alpha\beta}^\alpha \hat{\Gamma}_{\mu\nu}^\beta - \hat{\Gamma}_{\mu\beta}^\alpha \hat{\Gamma}_{\alpha\nu}^\beta, \quad (2.2)$$

whereas the scalar curvature is given by

$$\hat{R} = g^{\mu\nu} \hat{R}_{\mu\nu}. \quad (2.3)$$

We can derive the field equation of f(R) gravity in the Palatini formalism from the above action (2.1)

$$F(\hat{R}) \hat{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f(\hat{R}) = \kappa^2 T_{\mu\nu}, \quad (2.4)$$

where $F(\hat{R}) = \partial f(\hat{R}) / \partial \hat{R}$ and the matter energy momentum tensor is given as usual form

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (2.5)$$

From the above equations we can get the generalized Einstein equation

$$\hat{G}_{\mu\nu} = G_{\mu\nu} + \frac{3}{2} \frac{1}{F^2} \nabla_\mu F \nabla_\nu F - \frac{1}{F} \nabla_\mu \nabla_\nu F + g_{\mu\nu} \frac{1}{F} \square F - \frac{3}{4} g_{\mu\nu} \frac{1}{F^2} (\partial F)^2, \quad (2.6)$$

where $\hat{G}_{\mu\nu} \equiv \hat{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \hat{R}$ and $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$. For the later use, we reexpress the above equation again as

$$F G_{\mu\nu} = \kappa^2 T_{\mu\nu} - \frac{3}{2} \frac{1}{F} \nabla_\mu F \nabla_\nu F + \nabla_\mu \nabla_\nu F + \frac{1}{2} (f - F R) g_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \square F. \quad (2.7)$$

From the above equation we can derive 00-component of the modified Einstein equation

$$-3H^2 - 3HH' = \frac{\kappa^2}{F} \rho + \frac{3}{2} H^2 \left[\frac{F''}{F} + \left(1 + \frac{H'}{H} \right) \frac{F'}{F} - \frac{F'^2}{F^2} \right] - \frac{1}{2} \frac{f}{F}, \quad (2.8)$$

where primes denote derivatives with respect to $\ln a$. We also have the following useful equation

$$-2 \frac{H'}{H} = \frac{\kappa^2}{F H^2} (\rho + p) + \left[\frac{F''}{F} + \left(-1 + \frac{H'}{H} \right) \frac{F'}{F} - \frac{3}{2} \frac{F'^2}{F^2} \right]. \quad (2.9)$$

3 Linear perturbation in f(R) gravity (In Conformal Newtonian Gauge)

First we start from the metric in the conformal Newtonian gauge. The line element is given by

$$ds^2 = a^2(\tau) \left[-\left(1 + 2\Psi(\tau, \vec{x})\right) d\tau^2 + \left(1 - 2\Phi(\tau, \vec{x})\right) dx^i dx_i \right]. \quad (3.1)$$

The main modifications for viable models with stable high curvature limits happen well during the matter dominated epoch and we can take the components of the energy momentum tensor as

$$T_0^0 = -\rho(1 + \delta), \quad T_i^0 = \rho \partial_i q, \quad T_j^i = 0. \quad (3.2)$$

From the previous equation (2.7), we can find the perturbed Einstein equation

$$\begin{aligned} F \delta G_\nu^\mu &= \kappa^2 \delta T_\nu^\mu - R_\nu^\mu \delta F - \frac{3}{2} \frac{\delta(\nabla^\mu F \nabla_\nu F)}{F} + \frac{3}{2} \frac{\nabla^\mu F \nabla_\nu F}{F^2} \delta F + \delta(\nabla^\mu \nabla_\nu F) \\ &\quad + \left(\frac{3}{2} \frac{\square F}{F} \delta F + \frac{3}{4} \frac{\delta(\partial F)^2}{F} - \frac{3}{2} \frac{(\partial F)^2}{F^2} \delta F - \delta \square F \right) \delta_\nu^\mu, \end{aligned} \quad (3.3)$$

where we use the equation (7.7) and $\delta f(\hat{R}) = F(\hat{R}) \delta \hat{R}$. If we consider the ij -component of the perturbed equation, then we can find

$$\Phi - \Psi = \frac{\delta F}{F}, \quad (3.4)$$

where we assume the null anisotropic stress. If we use the above equation (3.4), then we can express the other components of the perturbed Einstein equation (3.3)

$$\begin{aligned} 3H^2 \left[\Phi' + \Psi' + \frac{1}{2} \frac{F'}{F} (\Phi' + \Psi') + \left(\frac{1}{2} \frac{F''}{F} - \frac{1}{2} \frac{F'^2}{F^2} + \frac{1}{2} \frac{H'}{H} \frac{F'}{F} + \frac{1}{2} \frac{F'}{F} + \frac{H'}{H} + 1 \right) \Phi \right. \\ \left. + \left(-\frac{1}{2} \frac{F''}{F} + \frac{F'^2}{F^2} - \frac{1}{2} \frac{H'}{H} \frac{F'}{F} + \frac{3}{2} \frac{F'}{F} - \frac{H'}{H} + 1 \right) \Psi \right] + \frac{k^2}{a^2} (\Phi + \Psi) = -\frac{\kappa^2 \rho}{F} \delta, \end{aligned} \quad (3.5)$$

$$H \left[\Phi' + \Psi' + \Phi + \Psi + \frac{1}{2} \frac{F'}{F} (\Phi + \Psi) \right] = -\frac{\kappa^2 \rho}{F} q, \quad (3.6)$$

$$\begin{aligned} 3H^2 \left[\Phi'' + \Psi'' + \left(4 + \frac{H'}{H} \right) \Phi' + \left(3 \frac{F'}{F} + 4 + \frac{H'}{H} \right) \Psi' + \left(-\frac{F''}{F} - \left(2 + \frac{H'}{H} \right) \frac{F'}{F} \right. \right. \\ \left. \left. + \frac{H'}{H} + 3 \right) \Phi + \left(3 \frac{F''}{F} + \left(6 + 3 \frac{H'}{H} \right) \frac{F'}{F} + 3 \frac{H'}{H} + 3 \right) \Psi \right] = 0. \end{aligned} \quad (3.7)$$

To capture the metric evolution, let us introduce two parameters as in the reference [11]: θ the deviation from ζ conservation and ϵ the deviation from the superhorizon metric

evolution

$$\zeta' = \Phi' + \Psi - H'q = -\frac{H'}{H} \left(\frac{k}{aH} \right)^2 B\theta, \quad (3.8)$$

$$\Phi'' + \Psi' - \frac{H''}{H'} \Phi' + \left(\frac{H'}{H} - \frac{H''}{H'} \right) \Psi = -\left(\frac{k}{aH} \right)^2 B\epsilon, \quad (3.9)$$

where we define the dimensionless quantity

$$B = \frac{F'}{F} \frac{H}{H'}. \quad (3.10)$$

From the above equations, we can find the expression for θ and ϵ ,

$$\begin{aligned} \frac{H'}{H} \left(\frac{k}{aH} \right)^2 B\theta &= \frac{1}{2} \left[\frac{B'}{B} + \frac{3}{2} \frac{H'}{H} B + \frac{H''}{H'} + 4 \right] (\Phi - \Psi) + \frac{1}{2} \left[2H' + \frac{\kappa^2 \rho}{FH} \right] q \\ &= \frac{1}{2} \left[\frac{B'}{B} + \frac{3}{2} \frac{H'}{H} B + \frac{H''}{H'} + 4 \right] (\Phi - \Psi) + \frac{1}{2} \left[-\frac{H'}{H} B' + \left(\frac{H'}{H} - \frac{H''}{H'} \right) B \right. \\ &\quad \left. + \frac{1}{2} \frac{H'^2}{H^2} B^2 \right] Hq \end{aligned} \quad (3.11)$$

$$\begin{aligned} \left(\frac{k}{aH} \right)^2 B\epsilon &= \left[\frac{H'}{H} B + \frac{H''}{H'} + \frac{H'}{H} + 3 \right] \Phi' + \frac{1}{2} \left[-2 \frac{B^2}{B^2} - \left(4 \frac{H''}{H'} + 6 \right) \frac{B'}{B} + \frac{1}{2} \frac{H'^2}{H^2} B^2 \right. \\ &\quad \left. + \frac{5}{2} \frac{H'}{H} B - 6 \frac{H''}{H'} - 2 \frac{H''^2}{H'^2} + 3 \frac{H'}{H} + 3 - 2 \left(\frac{H''}{H'} + \frac{H'}{H} + 3 \right) \frac{1}{B} \right] \Phi \\ &\quad + \frac{1}{2} \left[2 \frac{B^2}{B^2} + \left(4 \frac{H''}{H'} + 6 \right) \frac{B'}{B} + \frac{1}{2} \frac{H'^2}{H^2} B^2 + 2 \frac{H'}{H} B' + \left(2 \frac{H''}{H} + \frac{5}{2} \frac{H'}{H} \right) B \right. \\ &\quad \left. + 8 \frac{H''}{H'} + 2 \frac{H''^2}{H'^2} - \frac{H'}{H} + 3 + 2 \left(\frac{H''}{H'} + \frac{H'}{H} + 3 \right) \frac{1}{B} \right] \Psi \\ &= -\frac{1}{2} \left[2 \frac{B'^2}{B^2} + \left(5 \frac{H''}{H'} + \frac{H'}{H} + 9 \right) \frac{B'}{B} + \frac{H'}{H} B' + \frac{H'^2}{H^2} B^2 + \left(\frac{5}{2} \frac{H''}{H} + \frac{3}{2} \frac{H'^2}{H^2} \right. \right. \\ &\quad \left. \left. + 6 \frac{H'}{H} \right) B + \frac{H''}{H} + 3 \frac{H''^2}{H'^2} + 13 \frac{H''}{H'} + \frac{H'}{H} + 9 + 2 \left(\frac{H''}{H'} + \frac{H'}{H} + 3 \right) \frac{1}{B} \right] \\ &\quad (\Phi - \Psi) + \left[\frac{H'}{H} B' + \frac{1}{2} \frac{H'^2}{H^2} B^2 + \left(\frac{H''}{H} + \frac{3}{2} \frac{H'}{H} \right) B \right] \Psi - \frac{1}{2} \left[\frac{H'}{H} B + \frac{H''}{H'} \right. \\ &\quad \left. + \frac{H'}{H} + 3 \right] \frac{\kappa^2 \rho}{FH} q \end{aligned} \quad (3.12)$$

From equation (3.11), we can recover the conservation of Newtonian gauge when $\Phi = \Psi$ and F is a constant.

In addition to these equations, we can find very useful equation from the structure equation (2.4). By taking the trace of this equation (2.4) and differentiate with $\ln a$, we

have

$$\hat{R}' = \frac{\kappa^2}{F_{,\hat{R}}\hat{R} - F} T'. \quad (3.13)$$

Also by taking the perturbation of the equation we have

$$\delta F \equiv F_{,\hat{R}}\delta\hat{R} = \frac{F_{,\hat{R}}}{F_{,\hat{R}}\hat{R} - F} \kappa^2 \delta T. \quad (3.14)$$

From these two equations, we can find

$$\frac{\delta F}{F} = -\frac{1}{3} \frac{F'}{F} \delta = \Phi - \Psi. \quad (3.15)$$

From this equation we can find the evolution equation of matter density fluctuation

$$\begin{aligned} \delta'' + \left(2\frac{B'}{B} + \frac{H'}{H}B + 2\frac{H''}{H'} - \frac{H'}{H} + 3\right)\delta' + \left(\frac{H'}{H}B' - 2\frac{B'^2}{B^2} - \left[4\frac{H''}{H'} + 2\frac{H'}{H} + 4\right]\frac{B'}{B} \right. \\ \left. + \frac{H'^2}{H^2}B^2 + \left[\frac{H''}{H} - \frac{H'^2}{H^2} + 5\frac{H'}{H}\right]B + \left[-3\frac{H''}{H} - 4\frac{H''}{H'} - 2\frac{H''^2}{H'^2} + \frac{H'^2}{H^2} - 2\frac{H'}{H} + 6\right] \right. \\ \left. - 4\left[\frac{H''}{H'} + \frac{H'}{H} + 3\right]\frac{1}{B}\right)\delta = -3\Psi'. \end{aligned} \quad (3.16)$$

Compared with previous works [8], we do not specify the gauge of matter density to solve the matter density fluctuation.

4 Stability of metric fluctuations

Unstable metric fluctuations can create order unity effects that invalidate the background expansion history. We can derive the evolution equation of the deviation parameter. If we differentiate the equation (3.12) and consider the evolution in the superhorizon scale, then we have

$$\begin{aligned} \epsilon'' + \left(2\frac{B'}{B} + \frac{H'}{H}B + \frac{H''}{H'} - 3\frac{H'}{H} - 1\right)\epsilon' + \left(-2\frac{B'^2}{B^2} + 2\frac{H'}{H}B' - \left[5\frac{H''}{H'} + 4\frac{H'}{H} + 9\right]\frac{B'}{B} \right. \\ \left. + \frac{1}{2}\frac{H'^2}{H^2}B^2 + \left[2\frac{H''}{H'} - 4\frac{H'^2}{H^2} + \frac{3}{2}\frac{H'}{H}\right]B + Q' + \left[-2\frac{H''}{H'} - \frac{H'}{H} + 1\right]Q + 4\frac{H'^2}{H^2} + 6\frac{H'}{H} + 7 \right. \\ \left. - 4\frac{Q}{B}\right)\epsilon = \frac{1}{B}F(\Psi, \Phi, Hq), \end{aligned} \quad (4.1)$$

where we use equations (3.7) and (3.9) and $F(\Psi, \Phi)$ is the source function for the deviation ϵ and define Q as

$$Q = \frac{H''}{H'} + \frac{H'}{H} + 3. \quad (4.2)$$

The above equation is different from the metric formalism [11]. The stability of ϵ depends on the sign of the coefficient of the term proportional to ϵ . In the metric formalism ϵ is stable as long as $B > 0$. However, the stability is complicate and need to be checked for each model in the Palatini formalism.

4.1 A particular example : $f(\hat{R}) = \beta \hat{R}^n$

We demonstrate the general consideration of the previous subsection with a specific choice for the nonlinear Lagrangian, $f(\hat{R}) = \beta \hat{R}^n$, where $n \neq 0, 2, 3$. The background is simply described by a constant effective equation of state in this model. The Hubble parameter scales as $H^2 \sim a^{-3/n}$. Then it is easy to write it with its derivatives in terms of $\ln a$

$$\frac{H'}{H} = -\frac{3}{2n}, \quad \frac{H''}{H} = \left(-\frac{3}{2n}\right)^2. \quad (4.3)$$

Here the scalar curvature is $\hat{R} = 3(3-n)H^2/(2n)$. From this fact, we can also find the derivatives of F with respect to $\ln a$

$$\frac{F'}{F} = \frac{F''}{F'} = \frac{3(1-n)}{n}, \quad \frac{F''}{F} = \frac{F'''}{F'} = \left(\frac{3(1-n)}{n}\right)^2. \quad (4.4)$$

If we use above equations (4.3) and (4.4) into (3.12), then we find that the deviation from the superhorizon metric evolution is null, $\epsilon = 0$.

5 Metric and matter density evolutions

5.1 Superhorizon evolution

We consider the metric evolution in superhorizon sized, $k/(aH) \ll 1$. In this case, the anisotropy relation of the equation (3.11) becomes

$$\Phi - \Psi \simeq (B + A)H'q, \quad (5.1)$$

where A is given by

$$A = -\frac{B(2B\frac{H'}{H} + 5)}{\left(\frac{B'}{B} + \frac{3}{2}\frac{H'}{H}B + \frac{H''}{H'} + 4\right)}. \quad (5.2)$$

From the above equation (5.1), we can find the superhorizon evolution equation of Φ from the equation (3.9)

$$\Phi'' + \left(\frac{B'}{B} + 2\frac{H'}{H}B + \frac{H''}{H'} - \frac{H'}{H} + 4 - C\right)\Phi' + \left(\frac{B'}{B} + \frac{H'}{H}B + \frac{H''}{H'} + 3 - C\right)\Phi \simeq 0, \quad (5.3)$$

where C is defined as

$$C = \frac{1}{B+A+1} \left[\frac{B'}{B} + 2\frac{H'}{H}B + 2\frac{H''}{H'} - \frac{H'}{H} + 3 \right]. \quad (5.4)$$

If we use this equation (7.12), then we have the evolution equation of matter density (3.16)

$$\begin{aligned} & \left(1 + \frac{B}{B+A} \right) \delta'' + \left(2\frac{B'}{B} + \frac{H'}{H}B + 2\frac{B'A - BA'}{(B+A)^2} - \frac{H'}{H} \frac{B}{B+A} + 2\frac{H''}{H'} - \frac{H'}{H} + 3 \right) \delta' \\ & + \left(\frac{H'}{H}B' - 2\frac{B'^2}{B^2} - \left[4\frac{H''}{H'} + 2\frac{H'}{H} + 4 \right] \frac{B'}{B} + \frac{H'^2}{H^2}B^2 + \left[\frac{H''}{H} - \frac{H'^2}{H^2} + 5\frac{H'}{H} \right] B \right. \\ & \frac{B''A - BA''}{(B+A)^2} - 2\frac{(B'A - BA')}{(B+A)^2} \frac{(B' + A')}{(B+A)} - \frac{H'}{H} \frac{B'A - BA'}{(B+A)^2} - \left(\frac{H'}{H} \right)' \frac{B}{B+A} \\ & \left. + \left[-3\frac{H''}{H} - 4\frac{H''}{H'} - 2\frac{H''^2}{H'^2} + \frac{H'^2}{H^2} - 2\frac{H'}{H} + 6 \right] - 4\left[\frac{H''}{H'} + \frac{H'}{H} + 3 \right] \frac{1}{B} \right) \delta = 0. \end{aligned} \quad (5.5)$$

5.2 Superhorizon evolution in a particular example

Now we can check the evolution equations in the previous subsection in a particular case, $f(\hat{R}) \sim \hat{R}^n$. In this case, we can simplify the following quantities

$$B = 2(n-1), \quad A = -4(n-1) = -2B, \quad C = \frac{3}{2n} = -\frac{H'}{H}. \quad (5.6)$$

From this, we can also simplify the evolution equations (5.3) and (5.5)

$$\Phi'' + \frac{9-4n}{2n}\Phi' = 0, \quad (5.7)$$

$$\delta' = 0. \quad (5.8)$$

The evolution equation of the Newtonian potential has no terms proportional to Φ , thus $\Phi = \text{constant}$ is a solution to the equation. Also the matter density fluctuation has the same for as general relativity, $\delta = \text{constant}$.

5.3 Subhorizon evolution

For subhorizon scales where $k/aH \gg 1$, we can find the Poisson equation from the equation (3.5)

$$k^2(\Phi + \Psi) \simeq -\frac{\kappa^2 a^2 \rho}{F} \delta. \quad (5.9)$$

If we use equations (3.15) and (5.9), then we have

$$3\Psi \simeq \left(-3\frac{\kappa^2 \rho}{FH^2} \frac{a^2 H^2}{k^2} + \frac{F'}{F} \right) \frac{\delta}{2} \simeq \frac{F'}{F} \frac{\delta}{2}. \quad (5.10)$$

From this equation we can find

$$\Phi \simeq -\Psi. \quad (5.11)$$

We can differentiate (7.10) and use the above equation (5.11) to get

$$\begin{aligned} & \Phi'' + \left(\frac{B'}{B} + \frac{5}{2} \frac{H'}{H} B + \frac{H''}{H'} + \frac{H'}{H} + 6 \right) \Phi' - \left(\frac{B'^2}{B^2} + \left[2 \frac{H''}{H'} - 1 \right] \frac{B'}{B} - 2 \frac{H'}{H} B' \right. \\ & \quad \left. - \frac{9}{4} \frac{H'^2}{H^2} B^2 + \left[2 \frac{H''}{H} - 4 \frac{H''}{H'} - \frac{21}{2} \frac{H'}{H} \right] B + \left[\frac{H''^2}{H'^2} - \frac{H''}{H'} + 10 \frac{H'}{H} + 6 \right] \right. \\ & \quad \left. + \left[2 \frac{H''}{H'} + 2 \frac{H'}{H} + 6 \right] \frac{1}{B} \right) \Phi \simeq 0. \end{aligned} \quad (5.12)$$

If we differentiate the equation (5.10) and put into the equation (3.16), then we have

$$\begin{aligned} & \delta'' + \left(2 \frac{B'}{B} + \frac{3}{2} \frac{H'}{H} B + 2 \frac{H''}{H'} - \frac{H'}{H} + 3 \right) \delta' + \left(\frac{3}{2} \frac{H'}{H} B' - 2 \frac{B'^2}{B^2} - \left[4 \frac{H''}{H'} + 2 \frac{H'}{H} + 4 \right] \frac{B'}{B} \right. \\ & \quad \left. + \frac{H'^2}{H^2} B^2 + \left[\frac{3}{2} \frac{H''}{H} - \frac{3}{2} \frac{H'^2}{H^2} + 5 \frac{H'}{H} \right] B + \left[-3 \frac{H''}{H} - 4 \frac{H''}{H'} - 2 \frac{H''^2}{H'^2} + \frac{H'^2}{H^2} - 2 \frac{H'}{H} + 6 \right] \right. \\ & \quad \left. - 4 \left[\frac{H''}{H'} + \frac{H'}{H} + 3 \right] \frac{1}{B} \right) \delta = 0. \end{aligned} \quad (5.13)$$

5.4 Subhorizon evolution in a particular example

We can use the previous relation (5.6) into the evolution equations (5.12) and (5.13)

$$\Phi'' + \frac{3(3-n)}{2n} \Phi' + \frac{3(14n^2 + 19n - 36)}{4n^2} \Phi = 0 \quad (5.14)$$

$$\delta'' + \frac{3(2-n)}{2n} \delta' = 0 \quad (5.15)$$

Even though the superhorizon scale evolutions of Φ and δ are same to those of general relativity, the subhorizon scale evolutions of them show different behaviors from those of general relativity as expected [12].

6 Conclusions

We have analyzed the cosmological evolution of linear perturbations in Palatini f(R) gravity to see the stability of metric fluctuations. We have also considered the matter density fluctuation in the Newtonian gauge.

Compared with metric f(R) gravity, we have shown that the stability of metric fluctuations in the high redshift limit of high curvature is not simply expressed. We need to

check each model for the stability. However, we have found that the deviation from the superhorizon metric evolution is null for a specific choice of the nonlinear Einstein-Hilbert action, $f(\hat{R}) \sim \hat{R}^n$ and stability of this model is guaranteed.

We have investigated the evolution equations of Newtonian potential and matter density contrast in super and sub-horizon scales. In the specific model, superhorizon evolutions of Newtonian potential and matter density fluctuation are same to those of general relativity. However, subhorizon evolutions show the different behaviors from the general relativity case. This will give us the method to probe the possibility of any modification of gravity.

7 Appendix

From the equation (2.2) we can derive the Ricci tensor and the scalar curvature by using the metric relation

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2} \frac{1}{F^2} \nabla_\mu F \nabla_\nu F - \frac{1}{F} \nabla_\mu \nabla_\nu F - \frac{1}{2} g_{\mu\nu} \frac{1}{F} \square F, \quad (7.1)$$

$$\hat{R} = R - 3 \frac{1}{F} \square F + \frac{3}{2} \frac{1}{F^2} (\partial F)^2. \quad (7.2)$$

We can rewrite field equation (2.6) as the form of Einstein equation plus corrections

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} + (1 - F) R_{\mu\nu} - \frac{3}{2} \frac{1}{F} \nabla_\mu F \nabla_\nu F + \nabla_\mu \nabla_\nu F + \frac{1}{2} (f - R) g_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \square F. \quad (7.3)$$

We can check that the correction terms are covariantly conserved by using the useful relation $(\square \nabla_\nu - \nabla_\nu \square) F = \nabla^\mu R_{\mu\nu}$.

Also by taking the trace of the equation (2.4) we can find

$$6H^2 + 3HH' = \frac{\kappa^2}{2F} (-\rho + 3p) - \frac{3}{2} H^2 \left[\frac{F''}{F} + \left(3 + \frac{H'}{H} \right) \frac{F'}{F} - \frac{1}{2} \frac{F'^2}{F^2} \right] + \frac{f}{F}. \quad (7.4)$$

By differentiation of the equation (2.9) we can derive another useful equation

$$\begin{aligned} & \frac{F'''}{F} - 3 \frac{F'' F'}{F^2} + \left(3 \frac{H'}{H} + 2 \right) \frac{F''}{F} + \frac{3}{2} \frac{F'^3}{F^3} - \left(3 \frac{H'}{H} + \frac{9}{2} \right) \frac{F'^2}{F^2} + \left(\frac{H''}{H} + \frac{H'^2}{H^2} + 3 \frac{H'}{H} - 3 \right) \frac{F'}{F} \\ &= -2 \left(\frac{H''}{H} + \frac{H'^2}{H^2} + 3 \frac{H'}{H} \right) \end{aligned} \quad (7.5)$$

The perturbed equations for the Ricci tensor and the scalar curvature are obtained from the equations (2.2) and (2.3)

$$\delta \hat{R}_\nu^\mu = \delta R_\nu^\mu + \frac{3}{2F^2} \delta (\nabla^\mu F \nabla_\nu F) - 3 \frac{\nabla^\mu F \nabla_\nu F}{F^3} \delta F - \frac{1}{F} \delta (\nabla^\mu \nabla_\nu F)$$

$$+\frac{(\nabla^\mu\nabla_\nu F)}{F^2}\delta F+\left(-\frac{1}{2F}\delta(\Box F)+\frac{\Box F}{2F^2}\delta F\right)\delta^\mu_\nu, \quad (7.6)$$

$$\delta\hat{R} = \delta R - \frac{3}{F}\delta(\Box F) + 3\frac{\Box F}{F^2}\delta F + \frac{3}{2F^2}\delta(\partial F)^2 - 3\frac{(\partial F)^2}{F^3}\delta F. \quad (7.7)$$

If we differentiate the equation (3.6) and use the equation (3.7), then we have

$$\frac{3}{2}\frac{F'}{F}(\Phi'-\Psi')+\left[\frac{3}{2}\frac{F''}{F}+\left(\frac{9}{2}+\frac{3}{2}\frac{H'}{H}\right)\frac{F'}{F}\right]\Phi+\left[-\frac{5}{2}\frac{F''}{F}-\left(\frac{7}{2}+\frac{5}{2}\frac{H'}{H}\right)\frac{F'}{F}-2\frac{H'}{H}\right]\Psi=-\frac{\kappa^2\rho}{F}\frac{q'}{H} \quad (7.8)$$

If we use the equation (2.9) and adopt $Hq' = -\Psi$, then we can rewrite the above equation (7.8) as

$$\Phi'-\Psi'+\left[\frac{B'}{B}+\frac{H'}{H}B+\frac{H''}{H'}+3\right](\Phi-\Psi)-\frac{H'}{H}B\Psi=0. \quad (7.9)$$

We can find Φ' from the equation (3.6) and (7.9)

$$\Phi'=-\frac{1}{2}\left[\frac{B'}{B}+\frac{3}{2}\frac{H'}{H}B+\frac{H''}{H'}+4\right](\Phi-\Psi)-\Psi-\frac{1}{2}\frac{\kappa^2\rho}{FH}q. \quad (7.10)$$

From the equations (3.15) and (5.1) we can derive

$$3\Psi=\frac{B}{B+A}\delta'+\left[\frac{B'A-BA'}{(B+A)^2}-\frac{H'}{H}\frac{B}{(B+A)}\right]\delta. \quad (7.11)$$

If we differentiate this equation (7.11), then we have

$$\begin{aligned} 3\Psi' &= \frac{B}{B+A}\delta''+\left[2\frac{B'A-BA'}{(B+A)^2}-\frac{H'}{H}\frac{B}{B+A}\right]\delta'+\left[\frac{B''A-BA''}{(B+A)^2}\right. \\ &\quad \left.-2\frac{(B'A-BA')}{(B+A)^2}\frac{(B'+A')}{(B+A)}-\frac{H'}{H}\frac{B'A-BA'}{(B+A)^2}-\left(\frac{H'}{H}\right)'\frac{B}{B+A}\right]\delta. \end{aligned} \quad (7.12)$$

References

- [1] A. Conley et al. (The Supernova Cosmology Project), *Astrophys. J.* **644**, 1 (2006) [astro-ph/0602411] ; A. G. Riess et al. (High-z Supernova Search Team), *Astrophys. J.* **607**, 665 (2004) [astro-ph/0402512].
- [2] D. N. Spergel et al. (Wilkinson Microwave Anisotropy Probe), *Astrophys. J. Suppl. Ser.* **170**, 377 (2007) [astro-ph/0603449].
- [3] D. Huterer and M. S. Turner, *Phys. Rev. D* **60**, 081301 (1999) [astro-ph/9808133].
- [4] H. A. Buchdahl, *Mon. Not. Roy. Astron. Soc.* **150**, 1 (1970).

- [5] M. Ferraris, M. Francaviglia, and C. Reina, *Gen. Rel. Grav*, **14**, 243 (1982).
- [6] X-H. Meng and P. Wang, *Class. Quant. Grav.* **21**, 951 (2004) [astro-ph/0308031] ; *Phys. Lett. B* **584**, 1 (2004) [hep-th/0309062] ; *Class. Quant. Grav.* **22**, 23 (2005) [gr-qc/0411007] ; D. N. Vollick, *Class. Quant. Grav.* **21**, 3813 (2004) [gr-qc/0312041] ; G. Allemandi, A. Borowiec, and M. Francaviglia, *Phys. Rev. D* **70**, 043524 (2004) [hep-th/0403264] ; T. P. Sotiriou, *Phys. Rev. D* **73**, 063515 (2006) [gr-qc/0509029]. N. J. Poplawski, *Class. Quant. Grav.* **23**, 4819 (2006) [gr-qc/0511071] ; *Phys. Rev. D* **74**, 084032 (2006) [gr-qc/0607124] N. J. Poplawski, *Class. Quantum Grav.* **24**, 3013 (2007) [gr-qc/0610133]; S. Nojiri and S. D. Odintsov, *Int. J. Geom. Meth. Mod. Phys.* **4**, 115 (2007) [hep-th/0601213] A. Iglesias, N. Kaloper, A. Padilla, and M. Park, [arXiv:0708.1163]
- [7] M. Amarguioui, O. Elgaroy, D. F. Mota, and T. Multamaki, *Astron. Astrophys.* **454**, 707 (2006) [astro-ph/0510519]; B Li, K.-C. Chan, and M.-C. Chu, *Phys. Rev. D* **76**, 024002 (2007) [astro-ph/0610794]; M. S. Movahed, S. Baghran, and S. Rahvar, *Phys. Rev. D* **76**, 044008 (2007) [arXiv:0705.0889]
- [8] T. Koivisto and H. Kurki-Suonio, *Class. Quant. Grav.* **23**, 2355 (2006) [astro-ph/0509422]; B. Li and M.-C. Chu, *Phys. Rev. D* **74**, 104010 (2006) [astro-ph/0610486]; K. Uddin, J. E. Lidsey, and R. Tavakol, *Class. Quant. Grav.* **24**, 3951 (2007). [arXiv:0705.0232].
- [9] T. P. Sotiriou, *Phys. Lett. B* **645**, 389 (2007) [gr-qc/0611107];
- [10] X.-H. Meng and P. Wang, *Gen. Rel. Grav.* **36**, 1947 (2004) [gr-qc/0311019]; E. Dominguez and D. E. Barraco, *Phys. Rev. D* **70**, 043505 (2004) [arXiv:gr-qc/0408069]; Gonzalo J. Olmo, *Phys. Rev. Lett.* **95**, 261102 (2005) [arXiv:gr-qc/0505101]; *Phys. Rev. D* **72**, 083505 (2005) [arXiv:gr-qc/0505135, gr-qc/0505136]; T. P. Sotiriou, *Gen. Rel. Grav.* **38**, 1407 (2006) [gr-qc/0507027].
- [11] Y.-S. Song, W. Hu, and I. Sawicki, *Phys. Rev. D* **75**, 04404 (2007) [astro-ph/0610532];
- [12] P. Zhang, *Phys. Rev. D* **73**, 123504 (2006) [astro-ph/0511218]